THE MILITARY INVENTORY ROUTING PROBLEM WITH DIRECT DELIVERY

THESIS

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Abstract

The inventory routing problem examines the coordination of inventory management and transportation policies when implementing vendor managed inventory replenishment. Vendor managed replenishment is the business practice in which a vendor monitors the inventory levels of its customers and decides when and how much inventory to replenish at each customer location. The United States Army uses vendor managed inventory replenishment in combat situations to manage supply operations to lower-echelon organizations. The military variant of the stochastic inventory routing problem requires consideration of delivery failures due to hostile actions taken by non-friendly forces. The loss of delivery vehicles negatively impacts future resupply capability. We formulate a Markov decision process model of the military inventory routing problem, the objective of which is to determine an optimal unmanned tactical airlift policy for the resupply of brigade combat team elements in an Afghanistan-like combat situation using cargo unmanned aerial systems for delivery. Computational results are presented for the military inventory routing problem with direct deliveries; the computational example is based upon United States Army resupply situations that occur in Afghanistan deployments. Base case results indicate that unmanned aerial systems have the capability to successfully perform the brigade combat team resupply mission, depending on the dynamics of the threat situation. An experimental design is employed to determine the set of factors important in a more general context.

Key words: logistics, dynamic programming, Markov decision process, stochastic inventory routing, unmanned aerial vehicles

To my wife and son, this would not have been possible without your love, support, and sacrifice.

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I. Introduction

The inventory routing problem (IRP) and associated vehicle routing problem (VRP) are foundational problems within the construct of any problem where products must move between a set of locations. Typical IRP applications model the supply of commodities from a supplier to a set of customers. Formulations include single and multiple suppliers as well as single and multiple customers. Models range from shipping a single commodity to multiple commodities. A subproblem of the IRP is the VRP, a variant that models the mode in which the inventory moves from the supplier to a customer. The type, capacity, and number of vehicles available for transporting inventory are modeled in the VRP. Moreover, vehicles have constraints that typically restrict the distance traveled, time traveled, and number of customer allowable in a route.

The objective of both the IRP and VRP is usually a minimization of costs but can also appear as a maximization of revenue. The costs considered include shipping costs to the supplier and the customers, depending on the particular application. Holding costs may apply for both the supplier and customers as well. Depending on the motivating problem, the supplier may not have enough inventory to meet the demand of the customers, in which case a penalty function also appears in the formulation of the objective. The shortage may or may not be backlogged, causing a possible loss of revenue. The cost of shipping is also considered, usually determined at a per mile, trip, or leg basis, thus allowing for both fixed and variable shipping costs. For a heterogeneous vehicle fleet, different shipping costs apply depending on the vehicle type. These costs can account for fuel, maintenance, and drivers. In a homogeneous vehicle fleet formulation these costs are constant across the vehicle fleet.

Customer demand in the IRP may be either deterministic or stochastic. For deterministic demand, knowledge of the demand may be known *a priori* or revealed at the time the vehicle arrives to the customer. If the knowledge of demand quantity is known *a priori* then shortages are known at the onset of the problem. However, in the case when demand is revealed upon arrival at the customer a shortage may occur for that delivery. Depending on the application, different formulations address the action space available in this occurrence. For stochastic demand, the distribution of customer demand for the commodity is assumed to be known to the supplier, although the actual demand of the customer is not realized until after the vehicle arrives to deliver the commodity, possibly resulting in shortages. The method of addressing shortages at this point is the same as with the deterministic demand models.

The rest of this paper is organized in the following manner. We complete this chapter with a review of current formulations and problem classes within the IRP and VRP in the literature. A focused review of the vendor managed inventory replenishment (VMI) business practice appears in Section 1.1.2, and the section concludes with a description of the Military Inventory Routing Problem (MILIRP). Next, Chapter II provides a general background for our problem of interest and concludes with a basic problem description and motivating problem. In Chapter III we formulate our particular problem and provide a discussion of how we discretized the continuous region considered for our application, utilize the threat map to construct the arc lengths between the nodes, and solve the VRP subproblem. We develop our dynamic programming formulation in Chapter IV and solve a small instance of the MILIRP exactly. Our results and conclusions are presented in Chapter V.

1.1 Literature Review

A review of current literature concerning the IRP and VRP informs our developlent of the MILIRP. Attention is given to find solutions that had motivating problems in industries that behave similarly to the military. In particular, we research any formulations that dealt with vehicle breakdown or failure to complete a route. General purpose solution methodologies and algorithms are also explored to gather insight into good solution techniques.

1.1.1 Inventory and Vehicle Routing Formulations.

Chien *et al.* [2] formulate the IRP and VRP such that the supplier does not necessarily have enough inventory to meet the demand of all the customers. The problem has a single supplier node. Profit margins for each customer are utilized to decide the amount of inventory delivered to each customer. Vehicle routes are constructed so as to not limit the number of potential customers visited. Upper and lower bounds on the objective function value are determined by respectively solving the Lagrangian relaxation and employing a heuristic method. Both the VRP and IRP subproblems are solved to optimality, and then a subgradient search method is employed to determine good multipliers for recombining the subproblems. The VRP is a multiperiod problem that can be solved by decomposing the problem into single period problems that are solved and which collectively approximate the solution of the multiperiod problem. The IRP is solved by examining the inter-period inventory flow and using this to link the periods for the VRP. Since total demand is not necessarily met in this formulation, the authors assess a penalty for any unmet demand for each customer. The penalty value is then used to determine prioritization for the next period. Within each period, each vehicle can only make at most one trip, and no transshipment is allowed between customers.

While the VRP is a well studied problem, relatively little research is dedicated to modeling and solving problems that account for disruptions to the route. Mu *et al.* [18] consider the case when a vehicle breaks down and cannot complete its route. One can model the breakdown as a Dynamic/Real-time Vehicle Routing Problem (DVRP) wherein the breakdown is the real-time component, and the evolving changes to the schedule provide the dynamic considerations. The focus of Mu *et al.* [18] is the development of a metaheuristic that provides good solutions quickly due to the time sensitivity of the generation of a new schedule. Mu *et al.* [18] consider the case of capacitated vehicles without time windows for deliveries. In generating solutions, only one extra vehicle exists for use since breakdowns are rare and not likely to occur more than once in a day. When a breakdown occurs, vehicles enroute to a location complete the delivery before rerouting is considered; however, if a vehicle just completed a delivery and has not departed, then it is immediately available for rerouting. In determining the priority of the feasible solutions, the preferred solution should not include the use of the extra vehicle. The next priority goes to solutions that minimize total distance traveled by all vehicles for the remaining deliveries. If a breakdown occurs after the last delivery of the vehicle, then no changes occur to the vehicle routes. The modeling of the breakdown occurs in a single instance of the VRP. The formulation provided does not attempt to solve the problem over a time horizon. The time of the breakdown is within a one-day routing of deliveries by a single supplier to multiple customers.

Bertazzi *et al.* [1] focus on addressing an IRP that has stochastic demands and allows for stock-out. In constructing the problem, each customer applies an order-up-to-level policy. This formulation does not allow for backlogging, and any shortage between the demand and order-up-to level is considered a loss, resulting in a penalty. The problem solved considers only one vehicle that has a given capacity. The objective of the formulation is to minimize the expected cost of inventory, penalty, and routing over the time horizon. Bertazzi *et al.* [1] begin with a dynamic programming formulation and use it to build a hybrid rollout algorithm that constructs good solutions to the problem and addresses the curses of dimensionality. The rollout algorithm is a useful heuristic for solving deterministic and stochastic dynamic programs. The algorithm developed is a hybrid because it approximates the cost-to-go of the dynamic program by providing an exact solution to a mixed integer linear program (MILP) model. The MILP models the deterministic expectation of the stochastic distribution of the demands. In solving the MILP model, Bertazzi *et al.* [1] develop a branch-and-cut algorithm that outperforms existing expected value algorithms.

Coelho & Laporte [3] develop exact solutions to several classes of IRPs. The model used is a general formulation of the multi-vehicle IRP (MIRP) that assumes holding costs for both the supplier and customers per time period. The customers have an inventory capacity while the supplier can meet the demand of all customers over the time horizon and in each time period. This prevents the inventory level at the customers from dropping below zero and prohibits backlogging. Each vehicle is constrained to one route that contains a subset of the supplier's customers. Since several variations of the MIRP exist, Coelho & Laporte [3] identify the following variants as solvable by their formulation: 1) quantity consistency, 2) vehicle filling rate, 3) order-up-to level, 4) driver consistency, 5) driver partial consistency, and 6) visit spacing. Both heterogeneous and homogeneous fleets are considered. Coelho & Laporte [3] employ a general objective function and constraints that, for each of the problem instances mentioned, are ignored or take on the form required by the particular class of MIRP. Coelho & Laporte [3] develop a branch-and-cut algorithm that solves the general formulation of the previous MIRPs exactly by solving a linear program relaxation via a dual simplex approach. When a new best solution occurs, a solution improvement algorithm is inserted that attempts improvements by removing or including a customer in the routes.

The vehicle routing problem with stochastic demand (VRPSD) is an extension of the VRP that assumes customer demand follows a known distribution. Novoa & Storer [19] study this problem and model the customer demand as following a known distribution. The actual demand is not known until the vehicle reaches the customer and delivers supplies. A general solution methodology for the VRPSD assumes an *a priori* solution for the vehicle route. The route is followed and if the quantity in the vehicle cannot meet the customer demands then extra trips to the supplier occur until all demands are satisfied. The objective of the formulation is to minimize total route costs, including both planned routes and extra routes resulting from additional returns to the supplier. Novoa & Storer [19] formulate a dynamic program that decomposes the problem into multiple stages. The program builds the vehicle route as it visits customers and determines whether or not to return to the supplier or proceed to another customer based on the current capacity of the vehicle. Novoa & Storer [19] combine three methodologies in their solution method. First, they introduce a rollout algorithm. The rollout algorithm assumes a sub-optimal *a priori* solution, called the base sequence, that has a cost-to-go that can be easily approximated. A two-stage algorithm is implemented that assumes there are two more customers to visit and at each delivery the algorithm requires an update to the base sequence. The second method that Novoa & Storer [19] implement is a Monte Carlo simulation (MCS). In computing the two-stage rollout algorithm MCS is implemented to reduce the computational time of the algorithm as it calculates the expected cost-to-go for all possible two-stage routes of the remaining unvisited customers to include trips back to the supplier. The final contribution of Novoa & Storer [19] is the development of a heuristic solution method for the single vehicle VRPSD. The heuristic uses a stochastic set-partitioning model for the initial formulation to build the base sequence.

1.1.2 Vendor Managed Inventory Replenishment (VMI).

Vendor managed inventory (VMI) replenishment is a business practice in which a central vendor monitors the inventory level of its customers and determines the quantity of commodities delivered and when deliveries are made to the customers [13]. This centrality of decision-making at the vendor level provides a dynamic that influences the decision timeline and formulation of the IRP. The intent of VMI is to minimize the sum of inventory costs and transportation costs over the entire network. In conventional inventory management, customers keep track of their own inventory and determine when to place an order for delivery of resources. The supplier receives the orders from the customers and then prepares the orders for delivery using its vehicles.

The disadvantages of conventional inventory management include non-uniformity of order demand and customer misinterpretation of urgent order criteria [13]. The first disadvantage addresses the issue of orders arriving simultaneously from customers due to customers checking their inventories at the same time and then placing orders. The second issue accounts for customers biasing the priority of their order by inflating its importance. This prevents true identification of urgent orders. The advantages of VMI include the vendor controlling inventory at the customers and the delivery schedule [14]. By controlling the inventory, the vendor determines when to make deliveries and is able to direct a more uniform schedule of deliveries and better manage its vehicle fleet. Furthermore, the vendor determines the criteria for delivery and can identify emergency situations when inventory levels are critical. Customers benefit from VMI as well because the central control of inventory increases reliability of not having shortages and they no longer have to devote resources to inventory management [14].

The implementation of VMI enables the vendor to determine the demand history of the customers [13]. The vendor combines this with the knowledge of customer locations to determine low-cost routing and vehicle management in order to meet demand. Kleywegt *et al.* [13] suggest that such routing and vehicle assignment can include capacitated and uncapacitated vehicle loading. Successful implementation of VMI requires timely and accurate information regarding customer inventory [14].

Kleywegt *et al.* [13] consider an IRP with the special case of direct deliveries and a single commodity. They consider the customers as having a capacity, and the vendor as having a fleet of homogeneous vehicles with a known capacity. Each customer has a demand with a probability distribution known to the vendor. Kleywegt *et al.* [13] formulate the problem such that the vendor can measure the inventory level at all customers at any time and determine when and how much commodity to deliver. In the general case, delivery to multiple customers can be combined on a route; however, this is reduced to a direct delivery to a single customer on each route. The routes that Kleywegt *et al.* [13] construct can all be traversed in less than a day, which makes possible multiple deliveries to a customer in a day. This also means that the entire vehicle fleet is available at the beginning of every day. Kleywegt *et al.* [13] formulate the IRP as a Markov decision process (MDP) over an infinite time horizon and apply dynamic programming to solve the problem. They define a state as the current inventory at each customer; the action space to contain the set of decisions to deliver an amount of commodity to each customer; and the reward function to account for the revenue gained for positive difference between commodity required, commodity delivered, and commodity in inventory, minus the holding cost of left over inventory at the customer. A negative difference in inventory is discouraged by applying a penalty function to any shortages at the customers. The known probability function of demand enables a Markov transition function that governs the transition function from state to state [13]. The stochastic nature of the demand means that the functional equation is the maximization of the expected total discounted gain, i.e., revenue minus costs, for all customers over an infinite horizon.

The work conducted by Kleywegt *et al.* [14] extends the work of Kleywegt *et al.* [13] by relaxing the direct delivery constraint in the VRP subproblem. In this formulation of the VRP the available routes expand to also consider multiple customers per route. Vehicles only complete one route per day, all routes are completed in a day, and all vehicles are available for use each day. Customer daily demands are independent random variables that follow a known probability distribution. Each customer has a finite inventory capacity that is not constant between customers. Stochastic customer demands allow for shortages, and a penalty function is imposed. However, shortages are not backlogged and represent a total loss to the vendor. Because of their specific motivating problem, Kleywegt *et al.* [14] do not consider an inventory holding cost to the vendor, only to the customer. They also formulate this problem as having an infinite horizon given the nature of the industry they are modeling. The IRP is formulated as a discrete time MDP. The state of the system is the inventory at each customer at time t; the decision space is the set of inventory that is delivered to each customer at time t given that customers can be combined on a route given vehicle capacity; and the decision taken identifies the amount of inventory delivered to each customer. Kleywegt *et al.* [14] assume that no inventory is used by the customer between the time it is measured and when inventory is delivered by the vendor. A known Markov transition function results from the fact that the only stochastic element is the customer demand, which has a known probability distribution. The revenue for a positive net difference between commodity required, commodity delivered, and commodity in inventory less the holding cost of inventory still at the customer and any shortage penalty cost comprise the reward function. The functional equation for this problem is the maximization of the expected discounted gain for all customers over an infinite horizon.

1.2 The Military Inventory Routing Problem

The motivating problem of interest in this paper is the sustainment of an infantry brigade combat team (BCT) in a combat environment through the use of cargo unmanned aerial systems (CUAS). An austere combat environment, like that of Afghanistan, is of particular interest. Our purpose is to develop a dynamic programming formulation that exactly solves a small instance of the MILIRP. Our formulation prescribes an optimal delivery policy with which the United States Army (Army) is able to maximize the tonnage of supplies delivered by a fleet of homogeneous CUAS in order to meet the supply demands of an IBCT with combat outposts (COPs) dispersed throughout its area of operation (AO). Key to our formulation is the modeling of the possible destruction of the CUAS. Our optimal delivery policy must consider the number of available CUAS, the impact of losing the CUAS on future deliveries, and an environment with changing threat conditions that the IBCT encounters. This allows us to investigate changes in the decisions to deliver to COPs as the probability of successfully delivering to the COPs decreases. The ultimate end state of our research is to inform the Army as it determines the number of CUAS to have in a fleet of vehicles with the purpose of complete sustainment of an IBCT over a 12-month deployment while facing varying threat conditions.

The Army identifies sustainment as a key function that commanders must use in order to accomplish missions [9]. Moreover, the Army defines sustainment as the actions necessary to provide the freedom to act, extend the reach of missions, and enable prolonged operations. Sustainment is essential to military operations [9]. The Army identifies six principles of sustainment. The following four principles have particular relevance to this paper: 1) responsiveness, 2) simplicity, 3) economy, and 4) survivability. Responsiveness enables commanders the ability to set and maintain a tempo without fatigue, rotation of forces, and extending operational reach. Simplicity provides clear, concise, repeatable action that standardizes procedures and establishes a repeatable rhythm. Efficient economic allocation and prioritization of resources combine to achieve the greatest affect and create economy. This includes the use of contract services to make the most of the use of military resources. Survivability is the protection of personnel, materiel, and resources against hostile action while ensuring mission accomplishment [10].

In the Army hierarchy, the brigade combat team (BCT) is the organizational level in which self-contained operations are conducted. A BCT owns all required assets necessary for its operational mission set. The brigade ensures that all subordinate units have the requirements necessary for successful operations. The brigade support battalion (BSB) is the element responsible for ensuring that the subordinate units within the brigade have all necessary supplies [7].

The responsibility for planning sustainment operations for the BCT, which includes supply, belongs to the BCT supply officer and includes the operations officer, supply officer, and support operations officer of the BSB. The BSB has the responsibility for planning, preparing, and overseeing sustainment operations in the brigade AO [8]. This requires coordination with and monitoring subordinate units' supply requirements. Distribution management within the BSB synchronizes and prioritizes the execution of supply operations within the BCT. This is accomplished through the use of automated systems and regular reporting by subordinate units which enables the planning of sustainment operations for the BCT [8]. The central monitoring and distribution of supply inventory that Army logisticians utilize mirrors the civilian business practice of VMI. Like the civilian sector, this reduces wasted resources because the BSB has the ability to move supplies where they are needed using a limited capacity fleet of vehicles.

II. Background

2.1 Army Resupply

The Army conducts military operations in a variety of operational environments. These environments often contain harsh, rugged terrain that includes mountains, deserts, and jungles [11]. The Army expects to continue to operate in remote locations characterized by these conditions. As a result, Army logistics and sustainment operations must provide support across a dispersed area of operation (AO). In an independent study conducted for the Army, General Dynamics Information Technology [12] identifies the following five challenges to resupply missions: 1) demand for large quantities of supplies across area of operations, 2) effects of enemy threat and action, 3) weather, terrain, and poor infrastructure, 4) availability of distribution assets, and 5) flexibility to respond to changes in the operational environment.

Military resupply missions rely heavily on the use of ground lines of communications (GLOCs). However, the existance and quality of infrastructure as well as the potential harshness of the environment affect the utility of GLOCs for resupply. The lack of improved roadways prevents vehicular travel, and pack animals are often used to traverse the rugged terrain. Moreover, GLOCs are extremely vulnerable to attacks by improvised exploding devices (IEDs) and other anti-access technologies which restrict freedom of movement [12]. For example, the targeting of convoys along GLOCs with IEDs accounted for 65% of all U.S. deployed fatalities between November 2002 and March 2009, with 18% occurring during sustainment operations [12]. Host-nation assets have been used to provide resupply; however, this incurs added security risks and vulnerabilities that can delay the availability of the supplies or compromise their delivery [12].

The remoteness of most of the COPs in Afghanistan makes GLOCs difficult if not impossible to use. Air assets, i.e., helicopters, provide an alternative means for resupply. Due to the nature of austere operational environments, logisticians prefer to use such aerial assets. An Army simulation determined that approximately a quarter of division's supply requirements can be met by current air assets [12].

Limitations do exist for air resupply resulting from the terrain and to a greater extent, the weather. During harsh and extreme weather, pilots cannot fly due to limited visibility, especially at higher altitudes where lower air density negatively impacts helicopter responsiveness. Moreover, as the Army tries to provide more supplies through aerial means, the threat of man-portable air defense systems (MANPADS) becomes a greater concern. Helicopters are most vulnerable during landing and take-off at the COPs, often fly in pairs to mitigate the threat of MANPADS, and may even have armed escort aircraft. At times, even implementing risk mitigation techniques are not enough, and risk reduction occurs by denying resupply missions until the situation in the area improves.

Due to the high operational tempo (OPTEMPO), combat mission support for helicopters takes precedence over sustainment missions. This limits the number and availability of helicopters for resupply operations, causing both unpredictability in support for resupply and the possibility of last minute changes to asset allocation [12]. This lack of consistent availability of military helicopters led to the use of "Jingle Air", a group of contractors that operate helicopters in support of resupply missions. During a deployment to Afghanistan in 2008-2009, the 801^{st} BSB moved more supplies through these contracts than by Army and Air Force means combined [12]. However, these missions could only be conducted over eight hour shifts, during daytime, and under conditions within the limits of the instrumentation. Another limitation of using contractors to conduct resupply missions is the set of allowable supplies that the contractors may move. Therefore, the use of convoys along vulnerable GLOCs continues for intra-theater sustainment operations due to military helicopters meeting operational needs and the limitations of civilian contractors [12].

2.2 Role of the UAS

Current sustainment operations require aviation assets and ground convoys to successfully meet the supply needs of the COPs. As a way to mitigate the use of convoys and rotary aircraft, the Army is investigating the employment of rotary cargo unmanned aircraft systems (CUAS) to conduct the supply missions. The use of rotary CUAS has particular interest given most of the COPs do not have the physical space for a runway nor the ability to secure one. By introducing rotary CUAS, the required number of ground convoys decreases, allowing for a better utilization of combat and support units. Moreover, rotary CUAS do not have the same flight ceiling restrictions since cabin pressure does not matter. This enables the attainment of higher altitudes, reducing the MANPAD threat and possibly enabling shorter supply routes and quicker delivery of supplies to the COPs. General Dynamics Information Technology [12] reports that manned rotary craft suffer from adverse weather limitations that range from poor visibility constraints to dangerous flight conditions for pilots. In contrast, CUAS is ideal for these types of conditions because soldiers will not be affected by the flying conditions. This lack of limitation for CUAS widens the delivery window of supplies and enables commanders and logisticians to plan more sustainment missions than possible with manned rotary aircraft.

Sustainment missions only account for a portion of the mission requirements of Army Aviation. Additionally, the Army does not have enough vertical lift capability to meet demands. General Dynamics Information Technology [12] notes that 42% of the Army's Chinooks, its heavy lift helicopters, were deployed during 2008. With less than half of its total force available to combat commanders, mission prioritization becomes paramount. A survey of deployed soldiers reveals that only about 5.7% believed that helicopter requests for sustainment missions would be supported, with mission priority as the main reason given for the request denial [12]. The implementation of rotary CUAS dedicated to sustainment missions provides two benefits. First, it provides a dedicated sustainment platform and fleet of assets to meet the sustainment needs of the combat commander. This enables logisticians to have a known and fixed number of aircraft, i.e, CUAS, available for planning and utilization to meet sustainment needs. Second, CUAS logistic support frees up helicopters for other types of missions, in particular combat, personnel transport, and medical operations.

High operational tempo (OPTEMPO) and reliance on contractors causes variability, uncertainty, and a lack of responsiveness in sustainment mission support and execution. On the one side, operational requirements cause sustainment missions to be canceled and support to be denied. Contractor support requires long-term regimented planning and coordination. This limits flexibility and prevents the support of immediate and emergency supply requests. General Dynamics Information Technology [12] provides the following quote by an Infantry officer, "If the support is not anticipated (more than 72 hours out), then you are not getting the support. There is no immediate resupply." A dedicated CUAS capability controlled by the logisticians can help mitigate these issues. As previously stated, a dedicated supply capability eliminates the competing requirements for the aircraft and allows the heavy lift helicopters to focus on operational missions and the CUAS to focus on sustainment. A dedicated CUAS controlled by logisticians also mitigates the issue of responsiveness. While planning sustainment is required, due to the high OPTEMPO that combat units operate within, supply requirements will change unexpectedly. Military logisticians understand this and have the ability to change the sustainment mission plans accordingly to meet the immediate demands of the units without having to go through the protocols required by using contractor services [12].

Among the supplies moved across the battlefield, bulk liquids is the largest type that must be transported. As COPs grow in size and number, more fuel has to be transported. In November of 2007 it was noted that it required an entire division's heavy lift capability to supply one brigade's bulk fluid requirement [12]. In order to transport bulk fluid by air, all other missions would be suspended. This is not a realistic option and, as such, ground convoys on GLOCs are necessary to transport bulk liquids. As stated earlier, GLOCs are vulnerable to IEDs and small arms fire. Since most bulk liquids are fuel, this becomes highly problematic and dangerous for the ground convoys and helicopters. A CUAS capability alleviates the reliance on GLOCs and enables the heavy lift helicopters to conduct operational missions. Also, CUAS helps to distance personnel from the volatile liquids during transportation and makes transport safer [12].

III. Problem Definition

3.1 Problem Description

We consider the case where an Infantry Brigade Combat Team (BCT) is operating in an austere combat environment. The BCT is responsible for security in its AO and disperses its subordinate units in COPs throughout the AO. Figure 1 depicts an AO in Southeastern Afghanistan and shows how an BCT may locate the COPs within it. The BCT headquarters and BSB, represented by the black diamond, are both located in the center of the AO to facilitate distribution of supplies. The subordinate units are located at the COPs, small colored diamonds, throughout the AO in a manner consistent with the BCT's overall counterinsurgency mission. Supplies are distributed to the battalions by the BSB. Since battalions do not possess air assets, distribution to the COPs occurs centrally through the BSB using a fleet of CUAS located with the BSB. Supplies are distributed to F COPs by means of V identical CUAS systems. A location specific commodity storage capacity C_i exists at each of the COPs. Each CUAS is identical and has a capacity G. The demand u_{it} for each COP is stochastic in nature and varies from day-to-day, depending on the combat operations conducted. The amount of supplies delivered each trip taken by a CUAS is d_i, which specifies the supplies delivered to COP i during the trip. When utilizing rotary air assets, it is infeasible to combine multiple deliveries where more than one location must be visited. This imposes a direct delivery formulation of the routing constraint. Furthermore, it can easily be shown that any delivery that has less than a full load is dominated by delivery at full vehicle capacity. The direct delivery routing formulation results in the d_i for each CUAS equaling G. Time is modeled discretely with time periods $t = 0, 1, ..., T$. Inventory level X_{it} represents the inventory at COP i at time t. All round-trip routes used in the model can be traversed in less than one

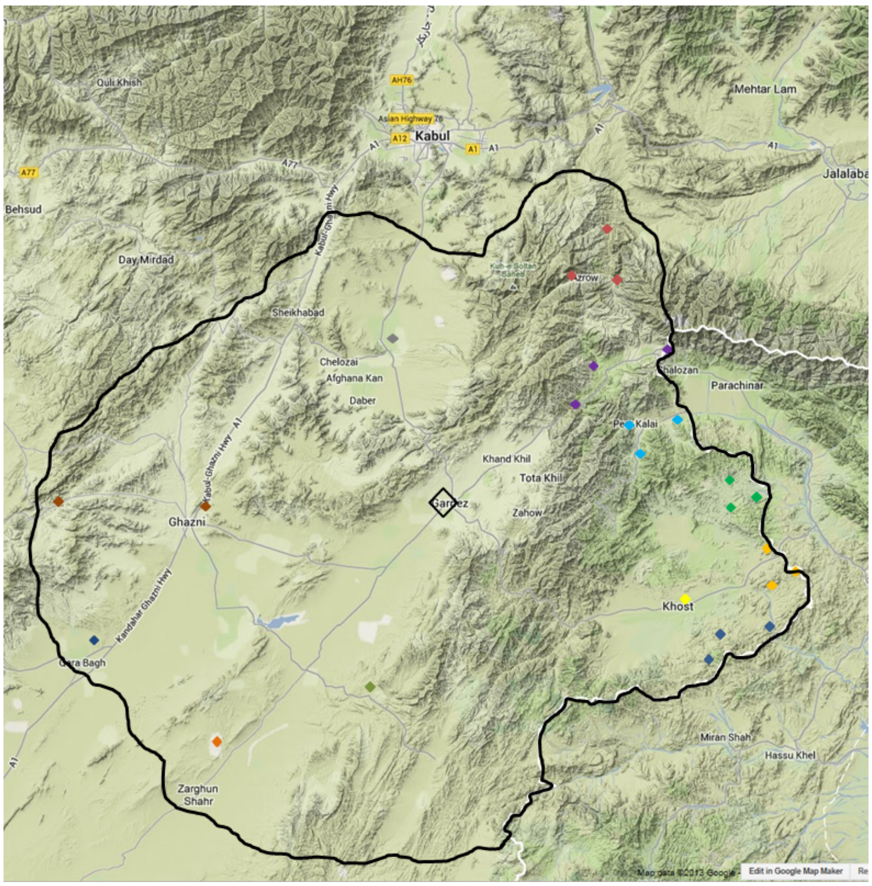

Figure 1. BSB and COP Locations throughout the IBCT AO

time period. The time required to execute a round-trip for a given route includes the time it takes to load the CUAS, travel to the COP, unload, return, and refuel. All of these activities are accomplished within one time period, which we assume is less than twelve hours. This enables multiple routes per day for each CUAS. It also means that all CUAS are available at the beginning of each day. For the range of the CUAS to not be an issue, the furthest direct delivery from the BSB to a COP must be within the operational radius of a fully loaded CUAS. Refueling occurs at the BSB depot at the end of a route, thereby mitigating the need to calculate maximum route lengths or legs before refueling. Time windows for deliveries are not considered in this problem since units are always available at the COPs to receive the supplies, and there is no other delivery location for the supplies, as the COPs represent a terminal node.

Most VRP and IRP formulations consider different costs associated with traversing a route or maintaining inventory levels. In our formulation, these costs are not of interest. While fuel, operation, and maintenance costs exist, these costs are not used to inform commanders in the decision of how to ensure supplies are delivered to their subordinates. Commanders are more concerned with the successful delivery to the subordinate units. For this reason, the cost of traveling a particular route is set aside in this research. Moreover, there is no cost for holding inventory. This occurs because there is no disincentive for units to have supplies in inventory. Shortages are considered in this model; however, a penalty function is not explicitly used to punish shortages. The reward function is the utility gained by delivering thousands of pounds of inventory by CUAS to each COP during time t. When a shortage occurs, we assume a ground convoy delivers enough inventory to bring the COP to capacity. This results in a lost opportunity of potential utility gained by having a

CUAS conduct the delivery and serves as the de facto penalty since the objective in this formulation is the maximization of utility gained by CUAS delivery.

3.2 Tessellation of the Geographic Region

In our formulation of the MILIRP we have an BCT with subordinate units dispersed through an area of operations. The brigade headquarters and its BSB are located near the region's geographic center to facilitate responsive support to all units. This location serves as the supplier or depot in our problem formulation, from where CUAS are dispatched to deliver supplies to subordinate units (i.e., customers). These subordinate units are located at COPs spread through the region's geography based on the tactical need for units to conduct operations. We restrict our attention to COPs within the maximum fully loaded radius of the CUAS, as measured via Euclidean distance.

In order to develop tractable instances to test our formulation of the MILIRP, we elect to tessellate a representative brigade-sized area of operations into finite elements using a mesh of uniformly-sized regular polygons. This discretization yields two benefits. First, it reduces the number of paths within the area of operations between the CUAS's base to a specific demand from an infinite set to a combinatorial set that depends on the granularity of the discretization. Second, it allows for the aggregation of risk to the CUAS in its flight over a discrete region based on enemy threat and challenging weather conditions (e.g., updrafts due to terrain).

For this study, we choose to discretize the brigade-sized area of operations using a mesh of uniformly-sized regular hexagons. Although square, rhombus, and triangular meshes are also feasible, Yousefi & Donohue [22] demonstrated that, for the same granularity of discretization, a hexagonal mesh exhibits the relative advantage of enabling clustering in more directions. In other words, a CUAS at one hexagonal cell has six directions from which to select its next location, whereas the cells resulting from a square (or rhombus) and triangular tessellation have only four and three, respectively. Thus, the hexagonal mesh discretizes the space into uniformly-sized and uniformly-shaped cells allowing for CUAS flight paths that more accurately represent the variability inherent in a continuous region. As in Lunday *et al.* [16], we use a horizontal orientation for our hexagonal mesh, wherein two opposing sides of the hexagonal cells lie on an East-West orientation. For our instances, we utilize regular hexagons with a side length of g equal to 3.69 km, resulting in cell areas of 9.59 km^2 each. Figure 2 illustrates the result of this tessellation, with the cell containing the

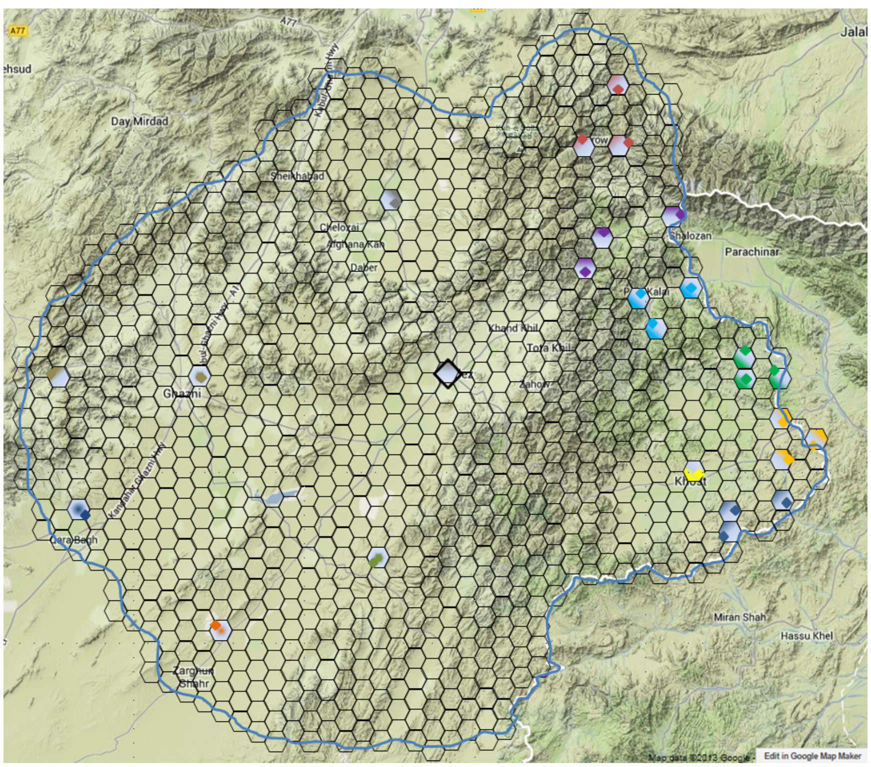

Figure 2. Tessellation of Geographic Region

BSB designated using a black diamond, and cells that contain COPs designated using shading and a colored diamond within to indicate the relative location of the COP.

3.3 Threat Map

When considering routing problems the military analyzes both the available paths and the threat of enemy action. The military identifies areas as having a certain threat level based on the probability of enemy action. For purposes of threat labeling, an area is represented by a hexagonal cell. The cumulative effect of the probabilities of enemy action at each leg within a route is used as a surrogate for determining the probability of successfully transiting between points on a route (i.e, between two adjacent hexagons). In this formulation, we consider two threat levels: high and low. We denote ω_l as the probability of successfully transitioning between two low threat areas. We denote ω_m as the probability of successfully transitioning between a low threat and high threat area in either direction. We denote ω_h as the probability of successfully transitioning between two high threat areas. Of note, the relationship between ω_l, ω_m, and ω_h is $\omega_h < \omega_m < \omega_l$.

The threat of enemy action is not constant over time. In a report by Cordesman *et al.* [4], the data of incidents of enemy attacks over the past three-and-a-half years in Afghanistan exhibit cyclic behavior. For our instances, we use this data to create a series of threat maps that model the probability of enemy action across the brigade area of operations. To increase realism, we group the threat maps by season and use them to characterize the change in the threat due to enemy action over the course of a year, with a total of M threat maps that approximate the threat conditions an BCT may encounter over a 12-month deployment.

Threat maps that represent the winter months have few high threat areas. The number of high threat areas increase until the summer months, which have the most high threat areas. The fall months illustrate a drop in high threat areas with a more marked drop going into winter. The transition between threat maps follows a Markovian transition function. Regardless of the current threat map, the transition to the next map is governed by a probability transition matrix Q that dictates the one-step transition probability to all other threat maps. This transition matrix itself changes given the current season; meaning transitions of threat maps characteristic of winter months to threat maps characteristic of summer months is less likely than transitions to other characteristically winter maps or spring maps. In determining which areas are high and low threat, we employ military terrain analysis techniques [6]. These techniques use an understanding of how terrain influences the movement of enemy forces and the locations where they concentrate their actions against friendly forces. BCTs conduct this type of analysis on a daily basis in order to determine operational missions as well as the routing of sustainment operations. Moreover, this analysis influences the actual probabilities implemented within the one-step transition matrix.

3.4 Routing Formulation

The vehicle routing problem (VRP) is a subproblem to the MILIRP. In our problem, the vehicles are CUAS and travel is conducted through the tessellated airspace. To limit the available routes, the airspace considered lies within the boundary of the BCT area of operations. A brigade controls all activity within its AO and deconflicts all movement. This mitigates the coordination and deconfliction of airspace as the BSB conducts the sustainment operations on behalf of the brigade. Internal deconfliction of airspace in this formulation assumes that routing based on threat of enemy action is the only factor and that friendly actions do not affect available routing options. Furthermore, routes are created by traversing adjacent hexagonal cells that form the tessellation mesh used to discretize the BCT AO. We solve instances of the VRP *a priori* and then populate the IRP and solve the dynamic program for the optimal policy.

We implement Dijkstra's algorithm on a network representation of the hexagonal cells that describe the BCT AO. Each hexagon represents an area to which a rotary CUAS can travel. In this formulation each hexagon is a node and is adjacent to at least one other node and no more than six nodes. The node network that represents the BCT AO is a fully connected, undirected graph and contains q nodes and g arcs. The threat level of each node pair in the network determines the arc distance between them; which represents the probability of successfully traversing the arc between the nodes in the network. For our problem, we assume the movement between nodes to depend only on the nodes being traversed and to be independent of any previous movement between nodes. This Markovian property is leveraged when determining the shortest path between any two nodes. The sequential movement between nodes can be expressed as the product of arc distances, p, in the route.

The multiplicative property of independent probabilities enables us to compute the probability of successfully traversing a route r from the BSB to a COP as:

$$\prod_{(j,k)\in r} p_{(j,k)}. \tag{3.1}$$

Note that finding the path r that maximizes Equation 3.1 is equivalent to finding the path r^* such that

$$r^* \in \arg\max \left\{ \prod_{(j,k)\in r} p_{(j,k)} \right\}, \tag{3.2}$$

which is also equivalent to finding the path:

$$r^* \in \arg\min \left\{ - \prod_{(j,k)\in r} p_{(j,k)} \right\}. \tag{3.3}$$

Applying a logarithmic transformation we obtain

$$r^* \in \arg\min \left\{ - \sum_{(j,k)\in r} ln(p_{(j,k)}) \right\}. \tag{3.4}$$

This allows us to denote the arc lengths as $-ln(p_{(j,k)})$ and utilize Dijkstra's Algorithm to identify the shortest path through such a network, which is equivalent to finding the path with the maximum probability of successful delivery of the commodity. We denote the probability of success for an outgoing route from the BSB to COP i using threat map m as ϕ_{im}. This is used in the dynamic programming formulation of the main IRP.

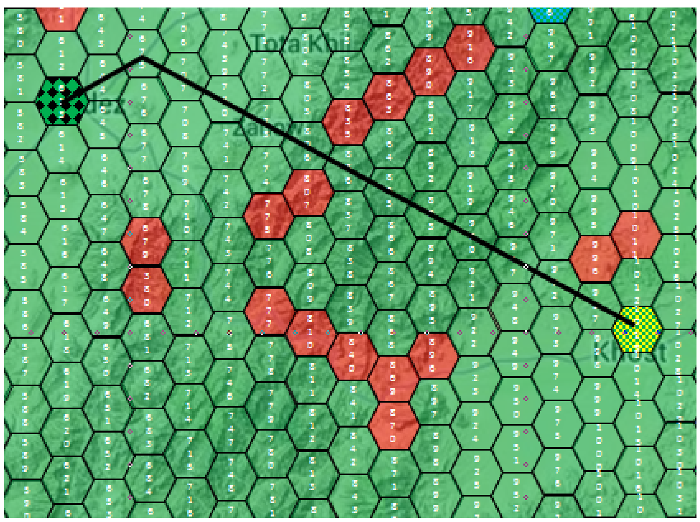

Figure 3. Route with Maximum Probability of Success for Delivery to COP 1 on Winter Threat Map 1

Dijkstra's algorithm provides an optimal route to each COP from the BSB for every threat map. As the areas of high threat change and the number of high threat areas change between each map, different routes are constructed. Figures 3 and 4 provide examples of implementing Dijkstra's algorithm in our problem. The low threat areas have hexagons that are colored green and the high threat areas are colored red. The BSB is a green and black checkered hexagon and has a low threat

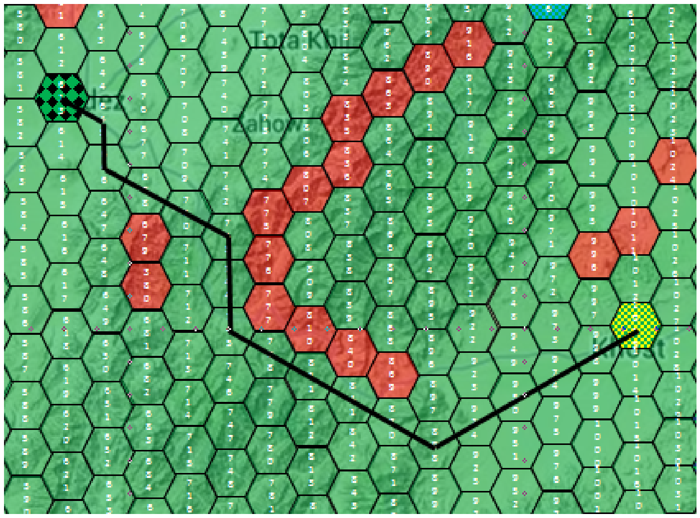

Figure 4. Route with Maximum Probability of Success for Delivery to COP 1 on Winter Threat Map 2

of enemy action. The COP is colored yellow and green and also has a low threat of enemy action. Figure 3 shows the route with the maximum probability of success from the BSB, in the top left, to COP 1, in the bottom right. The probability of success for this route is $\phi_{1,1}=(\omega_l)^{14}$ since 14 low-low arcs are traversed. Figure 4 shows the optimal probability of success route from the BSB to the same COP using a different threat map. As high threat areas change, the optimal probability of success route changes and is now $\phi_{1,2}=(\omega_l)^{17}$ due to traversing 17 low-low arcs. Figure 4 illustrates that the optimal route is longer yet has a higher probability of success than using the same route shown in Figure 3. If the same route is used as in Figure 3, then the probability of success for the route using the threat map from Figure 3 becomes $\phi_{1,2}=(\omega_l)^{12}(\omega_m)^2$,resulting in $\omega_{1,2} \ll \omega_{1,1}$.

The route distance that ϕ_{im} represents is the one-way distance from the BSB to COP i when on threat map m. In determining the round-trip distance, we assume that the threat map is the same for both legs of the total route. Using our previous example, for $\phi_{1,2}$ the complete round-trip distance from the BSB to COP 1 and back to the BSB is $(\omega_l)^{34}$.

3.5 Problem Formulation

We formulate the MILIRP as a finite horizon, discrete time MDP. The state space $\mathcal{S}$ is:
$\{0,1,...,C_1\} \times \{0,1,...,C_2\} \times ... \times \{0,1,...,C_F\} \times \{0,1,...,V\} \times \{1,2,...,M\}$. The inventory at COP i at time t is $X_{it} \in \{1,2,...C_i\}$. The number of CUAS at time t is $v_t \in \{0,1,...,V\}$. The threat level of the AO at time t is represented by threat map $m_t \in \{1,2,...,M\}$. Let $S_t = (X_{1t}, X_{2t}, ..., X_{Ft}, v_t, m_t) \in \mathcal{S}$ be the state at time t.

The action space $\mathcal{A}(s)$ for each state s is the number of CUAS, a_{it}, sent to COP i at time t. Let $A_t = (a_{1t}, a_{2t}, ..., a_{Ft}) \in \mathcal{A}(S_t)$ be a specific action taken at time t. The set of actions available at time t are governed by the following constraints

$$\sum_{i=1}^{F} a_{it} \leq v_t,\ \forall\ t \in \{1,2,...,T\}\,, \tag{3.5}$$

where $a_{it} \in \{0,1,...,v_t\}$. Furthermore, the number of CUAS that can be sent, during any period, to a specific COP, is constrained by the number of crews or the number of CUAS such that $a_{it} = \min\{v_t, crews\}$. Each CUAS carries its capacity G. When making a delivery to COP i the capacity of the COP, C_i, cannot be exceeded. Therefore, any potential overage is not delivered and returns to the BSB with the CUAS. For each COP i the actual amount of supplies delivered for a given decision is $d_i(a_{it})$. The capacity constraint of each COP is represented by $X_{it} + d_i(a_{it}) \leq C_i$ for each COP i and all times t. In order to account for supply consumption during time t let $U_t = (u_{1t}, u_{2t}, ..., u_{Ft})$ be the amount of supplies consumed at each COP at time t. The capacity constraint now becomes $X_{it} + d_i(a_{it}) - u_{it} \leq C_i$ for each COP i and all times t.

The probability of a CUAS successfully traversing a route from the BSB to COP i on threat map k is ϕ_{ik}. When a single CUAS is sent to a COP, there are three possible outcomes: 1) successfully traversing the route to and from the COP (SS), 2) successfully traversing the route to the COP and failing to complete the route on the way back to the BSB from the COP (e.g., shot down by enemy forces or suffering mechanical failure) (SF), and 3) failing to traverse the route while traveling to the COP (F). The probability of each of the outcomes is governed by ϕ_{ik}, which is obtained using Dijkstra's algorithm to find the path with the highest probability of successful completion of a route from the BSB to COP i given threat map k. Figure 5 illustrates the relationship between the outcomes and their associated probability of occurrence. The outcomes are fixed and have a known probability of occurrence that is dictated by the threat map. The probabilistic transition between threat maps is Markovian. The transition matrix Q is a square $M \times M$ matrix that describes the probability of transitioning between threat maps.

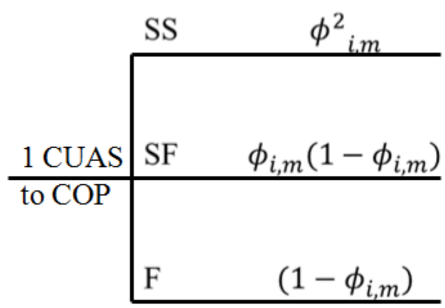

Figure 5. Probability of CUAS Outcome when Supplying a COP

When making a decision to send a_{it} CUAS to COP i the results $y_i = (y_{1i}, y_{2i}, y_{3i})$ are governed by a trinomial distribution $f_\phi(\vec{\mathbf{y}}|a_{it}, \eta_i)$,where

$$\eta_i = (\phi^2_{im}, \phi_{im}(1-\phi_{im}), (1-\phi_{im})) \tag{3.6}$$

denotes the probabilities of an SS, SF, or F event occurring. Let y_{1i}, y_{2i}, and y_{3i} denote the number of SS, SF, and F events that occur, respectively when resupplying

COP i. Consider the case in Table 1 where $a_{it} = 3$. The outcome $y_i = (2, 0, 1)$ in row three has a probability of $(\phi_{im}^2)^{y_{1i}} + (1 - \phi_{im})^{y_{3i}} = (\phi_{im}^2)^2 + (1 - \phi_{im})^1$ of occurring. Let

$$z_{it} = (y_{1i} + y_{2i}) \tag{3.7}$$

denote the number of CUAS that have a successful delivery.

Table 1. Example Outcome Space where $a_{it} = 3$

y_{1i}	y_{2i}	y_{3i}
3	0	0
2	1	0
2	0	1
1	2	0
1	1	1
1	0	2
0	3	0
0	2	1
0	1	2
0	0	3

The inventory at COP i at $t+1$ is related to the inventory at time t in the following manner:

$$X_{it+1} = \max\left\{X_{it} + d_i(a_{it}) - u_{it}, 0\right\}, \tag{3.8a}$$

with the additional consideration that

$$\text{if } X_{it+1} = 0, \text{then } X_{it+1} \leftarrow C_i. \tag{3.8b}$$

Because backlogging is not permitted, the inventory cannot be negative. Moreover, should the inventory fall to zero, then we assume a policy where a ground convoy is sent and delivers enough inventory to bring the COP to full capacity. This is not a desirable occurrence as no value is obtained by using a ground convoy. Furthermore, the actual amount of supplies delivered is stochastic and depends on a_{it} and η_i.

Delivery only occurs where z_{it} and a_{it} are positive. Therefore, $d_{it}(a_{it})$, the amount of supplies successfully delivered is $(z_{it})(G)$.

Similar to inventory, the number of CUAS at $t+1$ depends on the realization of a particular resupply mission outcome. Let l_{it} represent the number of CUAS destroyed in the course of delivering supplies to COP i at time t:(i.e., the sum of SF and F occurences)

$$l_{it} = (y_{2i} + y_{3i}), \tag{3.9}$$

The number of CUAS at $t+1$ resulting from delivering to COP i at time t is $v_{it+1} = v_{it} - l_{it}$. Therefore, the total number of CUAS at $t+1$ is

$$v_{t+1} = \sum_{i=1}^{F} v_{it+1}. \tag{3.10}$$

The single stage reward function for being in state S and taking action A at time t and then going to a state at $t+1$ is represented by $r_t(S_t, A_t, S_{t+1})$. The reward function is determined by the relationship:

$$r_t(S_t, A_t, S_{t+1}) = \sum_{i=1}^{F} d_i(a_{it}),\ \forall\ t \in \{1, 2, ..., T\}. \tag{3.11}$$

The objective function is to maximize the expected reward over all $t \in \{1, 2, ..., T\}$. Let $J_t(S_t)$ represent the expected reward gained over time interval $\{1, 2, ..., T\}$ for delivering supplies by CUAS. The value of $J_t(S_t)$ is calculated by solving the Bellman's equations where

$$J_t(S_t) \equiv \sum_{s' \in S} \{p(s'|S_t, A_t(S_t))[r_t(S_t, A_t, s') + J_{t+1}(s')]\}. \tag{3.12}$$

Let $J^*(s)$ be the optimal expected reward of state $s \in S_t$ where $J^*(S_t)$ equals the $J_t(s)$ that maximizes $J_t(S_t)$. Therefore, $J^*(S_t)$ solves the objective function by the following equation:

$$J^*(S_t) \equiv \max_{a_t(S_t)\in A(S_t)} \left\{ \sum_{s\in S} p(s|S_t, A_t(S_t))[r_t(S_t, A_t, s) + J_{t+1}(s)] \right\} \tag{3.13}$$

where $s \in S_{t+1} = (X_{1t+1}, X_{2t+1}, ..., X_{Ft+1}, v_{t+1}, m_{t+1})$.

3.6 Turnpike Methodology

The time horizon we consider in our formulation spans a year deployment of an BCT. Through the course of the deployment the BCT experiences all four seasons, each of which presents a different threat condition. We account for the differences between seasons by considering each as a separate instance and determine the proportion of supplies that can be delivered by CUAS in each season.

In a finite horizon, as one reaches the terminal epochs, maintaining the fleet of CUAS has little value when evaluating the decision of how many CUAS to use in delivering supplies during a high threat environment. The reward gained by delivering more supplies outweighs the need to retain CUAS for future deliveries at the end of the time horizon. However, reality dictates that from season to season and even at the completion of a deployment, retention of CUAS has value and meaning.

We implement a two-step method where we first solve for the turnpike, the optimal infinite-horizon decision rule, through value iteration and then implement the decision rule in the finite-horizon model [21]. Let Π be the set of optimal policies that represent the turnpike and let $\pi \in \Pi$ be a specific policy when in state S_t. The finite-horizon model has a horizon, t', such that $t' > t^*$, where t^* is the epoch in which the finite-horizon model reaches the turnpike policy, after which backward induction is used to determine the decisions for the remaining $t' - t^*$ epochs. At the first decision epoch, in which π is the optimal policy, consider in the next epoch that t' decision epochs still remain [21]. We determine the turnpike policy using an infinite-horizon model then fix this policy for employment in the finite-horizon model, in which we are most interested. This procedure is used so that we avoid risky CUAS delivery policies.

IV. Computational Example

4.1 One COP Scenario

A general formulation of the MILIRP is provided in Section 3.5. We now consider a specific instance of the BCT MILIRP where we solve for an ϵ-optimal solution to supplying a single battalion. We assume that the BCT delivers to a single COP where the battalion headquarters is located. Any subsequent distribution of supplies by the battalion (i.e., supplying the COPs of subordinate units) is not considered in this problem instance. In developing a solution that maximizes the amount of supplies delivered by CUAS over the course of a deployment, we discretize the time horizon and consider each season separately. This enables the development of seasonal solutions and takes advantage of the link between the seasons and the threat to CUAS.

4.2 Model Parameterization

We consider the simple case of one battalion sized COP. The consumption rate and the maximum amount of on-hand supplies determines the inventory capacity of the COP. General Dynamics Information Technology [12] indicates projected daily consumption of supplies by an infantry company is 29,928.2 lbs. An infantry battalion consists of a headquarters company, three infantry companies, and a weapons company [7]. The headquarters company and weapons company combined are generally the same size as an infantry company. Therefore, we model the battalion as having a daily requirement of four times that of an infantry company. In order to maintain consistency in discretization, we assume the battalion consumes 112,000 lbs of supplies daily. Furthermore, we fix the storage capacity to seven days of supplies with the consideration that the ability to hold a week's worth of supplies provides a conservative estimate of required capacity. Therefore, storage capacity C is 784,000 lbs.

We assume each season consists of 90 days. We further discretize time into epochs that divide a day into four six-hour time periods. This time resolution enables the modeling of day and night resupply operations. This is a valid assumption, as General Dynamics Information Technology [12] and the Department of the Army [11] both state that the implementation of CUAS enables the delivery of supplies in limited visibility situations. Furthermore, all round trip routes can be completed in less than six hours to include the time required to reload, refuel, and conduct preventative maintenance checks and services (PMCS) between routes. This also roughly matches the average flight hours associated with a "Jingle Air" mission of 6.4 hours [12].

In order to determine the value to use for the capacity of the CUAS, G, we consider the helicopters used by the Army and current CUAS efforts in the industry as benchmarks. The sling load weight of these helicopters is the value of interest as this is the capacity that can be attached underneath the helicopter and easily unloaded. Department of the Army [5] states that the maximum sling load weight of the UH-60A Blackhawk as 8,000 lbs. and the CH-47D Chinook as 26,000 lbs. Department of the Army [11] identifies 4,500 lbs. as the median load that CUASs should be able to handle from a design-and-capabilities perspective moving forward in development and procurement. General Dynamics Information Technology [12] recommends 4,000 lbs. as the load capacity as it is an even multiple of the standard pallet weights used by the Army. Furthermore, the average load of a "Jingle Air" helicopter is 4,000 lbs. [12]. Considering these requirements, results, and capabilities, we set G to 4,000 lbs. thereby allowing us to consider its capability as equal to civilian contractors. Using a 4,000 lbs. capacity for the CUAS enables scaleable capacity comparison with the Blackhawk and Chinook.

In determining the size of the CUAS fleet, we consider the minimum number of CUAS required to meet the daily demand of the battalion-size COP. The battalion requires 112,000 lbs. every four time periods. CUAS capacity of 4,000 lbs. means that a fleet of seven CUAS is required. The Department of the Army [11] considers a CUAS platoon to consist of two crews with a fleet of four CUAS. We apply this as the basic component for the fleet size in our formulation. Supplying a battalion requires the support of six platoons organized into two companies. Therefore, our vehicle fleet has 12 crews and 24 CUAS.

We consider the case where there are $M = 2$ threat maps. We use a scenario with two threat maps to enable the modeling of a low threat environment and a high threat environment. The transition matrix Q is a 2×2 matrix and has the form:

$$Q = \begin{vmatrix} \alpha & (1-\alpha) \\ (1-\beta) & \beta \end{vmatrix}. \tag{4.1}$$

The parameters α and β represent the probability of returning to the same threat condition within each season. Table 2 shows the values of α and β for each season, where α represents returning to a low threat map and β a high threat map. These values differ between seasons to reflect the changing nature of threat conditions throughout a year.

The exact value of ϕ_{im}, the probability of successfully traversing a route one-way, is not parametrized. Instead, we parameterize the probability of successfully traversing the tessellations within a route as: $\omega_l = 0.999$, $\omega_m = 0.950$, and $\omega_h = 0.900$. These values are used in Equation 3.4 to solve for the optimal route r^* that specifies the value of ϕ_{im}. We parameterize each ω to generate ϕ_{im} similar to that used in the study by General Dynamics Information Technology [12], in which CUAS have similar loss rates as Shadow UAS: approximately 0.022%.

Table 2. α **and** β **for each Season**

Season	α	β
Winter	0.95	0.25
Spring	0.85	0.30
Summer	0.65	0.90
Fall	0.80	0.50

Let $J^*_{IH}(S_t)$ be the optimal value of being in state S_t under steady-state conditions. The infinite-horizon model calculates $J^*_{IH}(S_t)$ by solving

$$J^*_{IH}(S_t) \equiv \max_{a \in A(S_t)} \left\{ E^{\pi}_{s} \left[\sum_{t=0}^{\infty} \gamma^t r(S_t, A_t) \right] \right\}. \tag{4.2}$$

The process of solving $J^*_{IH}(S_t)$ identifies a specific action $A'_t(s)$ to take when in state s. Let $\pi^*_{IH} = A'_t(s)$ for each s, be an element of Π, the set of policies. Let γ be the discount factor. Powell [20] offers $0 < \gamma \leq 0.7$ as an initial range to use for γ in order to improve performance. We let $\gamma = 0.9$. We implement a two tier stopping rule. First, we utilize the convergence test

$$||J^*_{IH}(S_t) - J^*_{IH}(S_{t-1})|| = \frac{\epsilon(1-\gamma)}{2 * \gamma}, \tag{4.3}$$

where we set th tolerance to be $\epsilon = 0.1$. Second, an upper bound of $N = 100$, is placed on the number of iterations to calculate $J(S_t)$. Upon reaching the stopping criteria, we employ a backward induction algorithm with a fixed policy π^*_{IH} from Equation 4.2 to solve the finite-horizon problem modeled in Equation 3.13.

4.3 Base Case Results

The results of our battalion-sized one COP formulation are presented in Table 3. For the winter and spring seasons, a fleet of 24 CUAS with 12 crews successfully delivers about 30% of the total demand. During the summer season, a fleet of CUAS only delivers 5.32% of the daily supply requirements. The reason for the low percentage of supplies delivered is the high probability of being in a high threat map and the corresponding low probability of successfully completing a route from the BSB to the COP. The ϕ_{im} for the high threat map m of each of these seasons is greater than the ϕ_{im} of the low threat map for the summer season. Therefore, delivery of supplies during the summer will always underperform the rest of the seasons, given the same initial conditions. During the fall, about 20% of the total demand can be met by CUAS. Let δ_ϕ be the difference in the values of ϕ_{im} that represent the low and high threat conditions within each season. The δ_ϕ for winter and spring is less than 0.003, while for summer and fall the value is approximately 0.1. This reflects that the conditions in the spring and fall seasons mirror the conditions of their respective preceding season. The steep changes from fall to winter and spring to summer represent how the nature of the threat condition changes abruptly during these times of the year. Furthermore, the large δ_ϕ for the summer and fall seasons represents a very low probability of successfully delivering supplies to the COP.

Table 3. Percent of Supplies Delivered by Season: Base Case

Season	$\phi_{1,1}$	$\phi_{1,2}$	% Supplies Delivered
Winter	0.9860	0.9830	31.32
Spring	0.9850	0.9830	29.06
Summer	0.9800	0.8834	5.32
Fall	0.9830	0.8854	19.92

The winter and spring seasons have the same optimal policy. Within these seasons the optimal policy does differ for low and high threat days. When there are zero to seven CUAS, send all CUAS available. When eight to 24 CUAS are in the fleet, send enough CUAS to meet the period demand regardless of inventory level. When a shortage exists, send all available up to reaching the point where the COP capacity is met. Then send only enough CUAS to bring the COP to capacity. This policy likely results from the high probability of being in a low threat map and the small difference in ϕ_{im} between the low threat and high threat maps.

The summer and fall seasons have the same optimal policy as the winter and spring when in the low threat map condition. During the summer, when in the high threat map, all CUAS are sent without delivering overage due to the capacity constraint. This occurs because the ϕ_{im} of the high threat map is not so low as to overly restrict sending CUAS to make deliveries considering the situation at the next time period will not likely improve given the high value of β. During the fall, when there are seven or fewer CUAS, no deliveries are attempted. When there are more than eight, as the number of CUAS increase then more are sent to create an inventory that prevents stock-out. As the number of CUAS approachs 24 then deliveries are made to bring the inventory up to capacity.

4.4 Sensitivity Analysis on α, β and Crews

The parameterization of α and β represents an assessment of the nature of the transition between threat environments within a season. However, these values are dynamic and are dictated by the activities of hostile forces. We conduct a sensitivity analysis of these parameters to gain insight on the nature of their influence on the ability to deliver supplies in a combat environment. Table 4 contains the results of the initial assessments of α and β as well as two additional parameterizations for each season. We consider the initial parameters to be the best representation of the values and an upper limit on the nature of the relationship between α and β. The other two parameterizations represent a lower-limit representation of the relationship and an assessment in the middle, respectively.

Table 4. Percent of Supplies Delivered by Season: Varying α and β

Season	α	β	$\phi_{1,1}$	$\phi_{1,2}$	% Supplies Delivered
Winter	0.95	0.25	0.9860	0.9830	31.32
	0.90	0.30	0.9860	0.9830	30.91
	0.85	0.35	0.9860	0.9830	30.52
Spring	0.85	0.30	0.9850	0.9830	29.06
	0.80	0.35	0.9850	0.9830	28.84
	0.75	0.40	0.9850	0.9830	28.63
Summer	0.65	0.90	0.9800	0.8834	5.32
	0.70	0.85	0.9800	0.8834	5.99
	0.75	0.80	0.9800	0.8834	6.85
Fall	0.80	0.50	0.9830	0.8854	19.92
	0.75	0.55	0.9830	0.8854	19.07
	0.70	0.60	0.9830	0.8854	17.39

During the winter and spring season, regardless of the values for α and β, a fleet of CUAS still delivers around 30% of the supplies required. The percent of supplies delivered by CUAS does decrease some, but the difference between the maximum and minimum amounts is only 1%. We investigate the optimal policy for the winter and spring seasons to determine why the percentage of supplies delivered by CUAS does not significantly change. In general, when more CUAS exist than crews, the optimal policy follows a structure where, as inventory increases the number of CUAS sent to deliver supplies decreases from maximum number of crews available to enough CUAS to bring the inventory up to the storage capacity without delivering an overage of supplies. During the winter and spring seasons the difference between α and β ranges between 0.35 and 0.70 with α always greater than β. This large difference indicates that the high probability of recurrence of a low threat map and low probability of recurrence of a high threat map, coupled with the high ϕ_{im} of the high threat map, makes dilivering supplies advantageous even on a high threat map. The high threat map occurs infrequently enough that sending CUAS on those days is worth the risk of being destroyed by enemy action. The high values of ϕ_{im} make successful delivery of supplies very certain; therefore, even as α decreases and β increases, overall delivery is not affected very much. This indicates that, for high values of ϕ_{im}, the probability of remaining in a threat map is low enough that it does not significantly affect the percent of supplies delivered by CUAS.

The summer and fall seasons experience similar change in the delivery of supplies. During these seasons, as α increases and β decreases, 1% more supplies are delivered. The optimal policy on the low threat map is the same policy as that used during the winter and spring seasons. During high threat map conditions, summer delivery policy remains the same as in low threat map conditions: send CUAS to supply up to COP capacity. The high threat condition optimal policy for the fall season differs from the summer's. In the fall, when there are eight CUAS or more, supplies are delivered such that, as the number of CUAS increases, more deliveries are made even when inventory approaches capacity. When the number of CUAS are close to eight, then CUAS are sent only until inventory has a small buffer to prevent stock-out. As the number of CUAS increases, so do the number of deliveries even as inventory increases toward capacity. When there are seven or less CUAS, then no CUAS are sent to deliver supplies. The difference betwen α and β during the summer and fall seasons ranges between 0.10 and 0.30 where, during the summer, β is always greater than α, and in the fall the reverse is true. The probability of remaining in the current threat map is at least 0.50 in both seasons. This condition and the low ϕ_{im} of the high threat map indicate that the value and relationship of α and β significantly affect the percentage of supplies delivered by CUAS. When β is great than α, low delivery occurs, as in the summer. When α is greater than β, then even with a low ϕ_{im}, successful delivery of supplies approaches that of when ϕ_{im} is high. The higher percent of supplies delivered by CUAS in the fall being close to that of winter and spring is evidence of this case.

We also consider the case where the BSB has the ability to surge and has more than 12 crews available to send more CUAS during a single time period in order to deliver supplies. Given the structure of the units we consider the cases where 16 and 18 crews are available. Table 5 shows the results of this analysis, given the α and β values used in Table 3. The results show that, no matter the number of additional crews available, the percent of supplies delivered by CUAS does not change. This indicates that ϕ_{im}, α, and β have a stronger influence on the ability to deliver supplies than the number of crews. As previously stated, as long as the number of CUAS meets or exceeds seven, then the daily supply requirement can be met and stock-out avoided.

Table 5. Percent of Supplies Delivered by Season: Varying *crew*

Season	*crew*	α	β	% Supplies Delivered
Winter	12	0.95	0.25	31.32
	16	0.95	0.25	31.32
	18	0.95	0.25	31.32
Spring	12	0.85	0.30	29.06
	16	0.85	0.30	29.06
	18	0.85	0.30	29.06
Summer	12	0.65	0.90	5.32
	16	0.65	0.90	5.32
	18	0.65	0.90	5.32
Fall	12	0.80	0.50	19.92
	16	0.80	0.50	20.85
	18	0.80	0.50	20.85

4.5 Design of Experiment

4.5.1 Design.

The case presented in Section 4.1 represents a particular instance of the MILIRP. To understand the influences on the percent of supplies delivered by CUAS we develop a design of experiment. We use a central composite design consisting of a fractional-factorial design, a center run, and axial runs. For the fractional-factorial we implement a one-half fraction 2^{k-p} resolution VI design where k is the number of factors that we believe significantly effect the response, the percent of supplies delivered by CUAS, and p is the number of independent effects to be confounded, and two signifies the number of levels used to analyze each factor. A resolution R design is a design where no p-factor effect is aliased with another effect consisting of less than $R - p$ factors [17]. For this design, we use a design model with $k = 6$ factors and $p = 1$ with each factor having two settings: high and low. Furthermore, the design is resolution VI because the minimum length alias effect of a one-factor effect has five factors, whereby $R = 6$. The factors we consider to have a significant effect on the proportion of supplies delivered are: 1) ϕ_l, the threat level of the low threat map, 2) δ_ϕ, the difference between the threat level of the low threat map and high threat map, 3) V, the total number of CUAS, 4) α, the one-step probability of returning to the low threat map, 5) β, the one-step probability of returning to the high threat map, and 6) the number of *crew* that control the CUAS, where one crew operates one CUAS. The high and low settings of the factors as well as the center-run and axial values used appear in Table 6. The axial and center-runs allow us to test for curvature in the response to determine if a polynomial regression describes the percentage of supplies delivered by CUAS better than a linear regression. The axial-run modifies a center run with an extension in one direction for a single factor. Axial runs for each extreme of each factor are conducted. We include only one center-run because our Markov decision process is deterministic. Therefore, the response from multiple center-runs will be the same value and will not provide any additional indication of curvature than measurable from a single run.

We base the parameterization of our factor settings on reports and studies sponsored by the Army and on the conditions we desire to model. General Dynamics

Table 6. Factor Settings for 2^{6-2}_{VI} One-Quarter Fractional Factorial Design

Factor	-1.7244	-1	0	1	1.7244
ϕ_l	0.9810	0.9848	0.9900	0.9952	0.9990
δ_ϕ	0.016	0.03	0.05	0.07	0.084
α	0.51	0.61	0.75	0.89	0.99
β	0.02	0.22	0.5	0.78	0.98
crew	$0.39V$	$0.48V$	$0.60V$	$0.72V$	$0.81V$
V	8	15	20	25	28

Information Technology [12] conducts a business analysis of different scenarios with CUAS and other modes of supply delivery. This analysis considers the CUAS to behave similar to the Shadow UAS with aircraft loss rates of 0.022% and weather related impacts on mission of 3.4%. Using this data to bound our ϕ_l and δ_ϕ, we set the upper bound on ϕ_l based on the aircraft loss rate and set the lower bound such that the lower bound of ϕ_l and lower bound on δ_ϕ approximate the affect of weather. The lower bound of δ_ϕ also allows us to model the condition where there is little change in the overall threat level after the transition governed by Q. The upper bound on δ_ϕ allows us to model high threat environment, where 0.91 is the probability of successfully traversing a route to the COP. The high and low levels we use for α and β enable the modeling of a wide range of conditions. The upper bound of 0.89 for α models the condition where a return to the current threat condition is very likely. Similarly, using 0.61 indicates a moderately low probability of staying in the current threat condition. The value of 0.78 for the high level of β represents a moderately high likelihood of returning to a high threat environment. The low value of 0.22 for β reprents a infrequent recurrence to a high threat environment. The high and low setting values for α and β provide a buffer for the axial runs to test the extreme cases for each factor while maintaining useful parameter values that have meaning for application. The value for the low factor setting of V comes from adjusting to allow the lower axial value to maintain the minimum required CUAS to satisfy u_{it}. The upper bounds for these factors come from the Department of the Army [11], where a platoon of CUAS consists of four aircraft and two crews. We consider the case where two companies of three platoons each exist with the mission to support the supply needs of an infantry battalion. We set the high level number of CUAS as 25 to bracket this configuration. We consider *crew* as a proportion of V, which implies it is a continuous factor. However, only whole crews are feasible; therefore we round the value of *crew* when using it in the model.

The central composite design we implement contains a one-half fraction design from Montgomery [17] that uses 32 runs for the entire analysis instead of 64 runs required for a full 2^6 factorial design. The fractional design provides a more efficient use of runs and allows for the estimation of the factor effects. Furthermore, we conduct 12 axial-runs with a normalized distance of 1.7244 units from the center value for each factor and a single center-run. The entire central composite design uses 45 runs to test for the presence of curvature in the response, which indicates a second-order or higher polynomial model. The central composite design we implement appears in Table 7. The design in Table 7 is completely orthogonal. Therefore, if one or several of the design factors do not significantly affect the percentage of supplies delivered by CUAS, then removing those factors from the model will not affect the estimated effects of the other factors.

4.5.2 Results and Analysis.

We utilize the results of the design of experiment in Subsection 4.5.1 to conduct a factor screening experiment. The design enables a total of 21 main factors and two-factor interactions effects and six quadratic effects to be estimated. The screening experiment identifies which of these factors significantly influence the percent of supplies delivered by CUAS. In this analysis we consider a P-value of ≤ 0.05 as sig-

Table 7. Factor Settings for 2^{6-2}_{VI} One-Half Fractional Factorial Design with Center-Runs

Run	$crew$	β	α	δ_ϕ	ϕ_l	V
1	1	1	1	1	1	1
2	-1	1	1	1	1	-1
3	1	-1	1	1	1	-1
4	-1	-1	1	1	1	1
5	1	1	-1	1	1	-1
6	-1	1	-1	1	1	1
7	1	-1	-1	1	1	1
8	-1	-1	-1	1	1	-1
9	1	1	1	-1	1	-1
10	-1	1	1	-1	1	1
11	1	-1	1	-1	1	1
12	-1	-1	1	-1	1	-1
13	1	1	-1	-1	1	1
14	-1	1	-1	-1	1	-1
15	1	-1	-1	-1	1	-1
16	-1	-1	-1	-1	-1	1
17	1	1	1	1	-1	-1
18	-1	1	1	1	-1	1
19	1	-1	1	1	-1	1
20	-1	-1	1	1	-1	-1
21	1	1	-1	1	-1	1
22	-1	1	-1	1	-1	-1
23	1	-1	-1	1	-1	-1
24	-1	-1	-1	1	-1	1
25	1	1	1	-1	-1	1
26	-1	1	1	-1	-1	-1
27	1	-1	1	-1	-1	-1
28	-1	-1	1	-1	-1	1
29	1	1	-1	-1	-1	-1
30	-1	1	-1	-1	-1	1
31	1	-1	-1	-1	-1	1
32	-1	-1	-1	-1	-1	-1
33	0	0	0	0	0	0
34	-1.7244	0	0	0	0	0
35	1.7244	0	0	0	0	0
36	0	-1.7244	0	0	0	0
37	0	1.7244	0	0	0	0
38	0	0	-1.7244	0	0	0
39	0	0	1.7244	0	0	0
40	0	0	0	-1.7244	0	0
41	0	0	0	1.7244	0	0
42	0	0	0	0	-1.7244	0
43	0	0	0	0	-1.7244	0
44	0	0	0	0	0	-1.7244
45	0	0	0	0	0	-1.7244

nificant. The following seven factors significantly influence the response: ϕ_l, V, α, β, δ_ϕ, $\phi_l * \alpha$, $\phi_l * \beta$, $\alpha * \beta$, $\alpha * \alpha$, and $\delta_\phi * \delta_\phi$. The sixth, seventh, and eighth factors are two-factor interactions and the last two are quadratic factors. The exclusion of *crew* in the model raises an interesting question. Inherently, as the number of *crew* increases so does the number of CUAS available to deliver supplies at any given time; however, results indicate that *crew* does not impact the percent of supplies delivered by CUAS significantly. The exclusion of *crew* from the model begins to make sense given that for fixed values of α and β, changing *crew* did not change the percentage of supplies delivered by CUAS. Table 8 summarizes the results of the screening experiment and regression model generated by the factors included in the model.

Table 8. Factor Influence on Percent of Supplies Delivered by CUAS

Factor	Sum of Squares	% Contribution	P-Value
α	1309.5966	22.2049	< 0.0001
β	1171.8885	19.8700	< 0.0001
ϕ_l	1046.0373	17.7361	< 0.0001
V	652.3636	11.0611	< 0.0001
$\alpha * \phi_l$	286.2028	4.8527	0.0002
$\beta * \phi_l$	266.6895	4.5219	0.0004
$\alpha * \beta$	202.8098	3.4387	0.0015
$\delta_\phi * \delta_\phi$	177.0541	3.0020	0.0027
δ_ϕ	114.0517	1.9338	0.0139
$\alpha * \alpha$	95.3271	1.61632	0.0235

Table 8 provides several interesting results. First, the main factors account for 72.8% of the error in the model. This indicates that the factors α, β, ϕ_l, and V exhibit the most influence over the actual percentage of supplies delivered by CUAS during a season. The two-factor interactions explain 12.8% of the error with quadratic effects explaining only 4.6%. These are the combinations of factors used to describe the threat environment and indicates that the inter-dynamics of the environment is not as important as the individual components. The low quadratic factor explanation of error represents small curvature in the percent of supplies delivered by CUAS due to compounding a single factor's effect.

Table 9. Effects Model for Percent of Supplies Delivered by CUAS

Summary of Fit			
R^2	0.9024		
Adjusted R^2	0.8737		
Parameter Estimates			
Term	*Estimate*	*Lower 95%*	*Upper 95%*
α	5.8746	4.5170	7.2322
β	-5.5572	-6.9148	-4.1995
ϕ_l	5.2503	3.8927	6.6079
V	4.1462	2.7886	5.5038
$\delta_\phi * \delta_\phi$	3.1641	1.1754	5.1527
$\alpha * \phi_l$	2.9906	1.5112	4.4690
$\beta * \phi_l$	-2.8869	-4.3653	-1.4085
$\alpha * \beta$	-2.5175	-3.996	-1.0391
$\alpha * \alpha$	2.3217	0.3330	4.3103
δ_ϕ	-1.7336	-3.0912	-0.3760

The model we develop using the factors previously discussed provides a quadratic regression model with the parameters found in Table 9. This model explains 90.24% of the error in the percent of supplies delivered by CUAS. Furthermore, the difference in the R^2 and Adjusted R^2 values is small. This indicates that only useful factors are in the model [15].

We develop the regression model not as a prediction model but to gain insight into the factor effects on the percentage of supplies delivered by CUAS. The estimates for each factor are the coefficients in the regression model and indicate the change in the response over the range of the factor. Since each factor has two levels, the effect of the factor is twice the coefficient estimate.

The most significant factor, α, represents an 11% increase in the ability to provide supplies by CUAS. As the probability of remaining in a low threat environment increases, there are more low threat periods enabling more successful deliveries of supplies from the BSB to the COP. Likewise, β has a negative effect on the percentage of supplies delivered by CUAS of 11%. When the probability of remaining in a high threat condition increases, the ability to deliver supplies decreases as the probabiliy of successfully traversing a route between the BSB and COP deminishes. While these factors offset the other's effect in magnitude, α and β have an interaction affect that has a negative effect of 5% on the percent of supplies delivered by CUAS. This means that as α and β increase, the percentage of supplies delivered by CUAS decreases. This shows that even when the probability of returning to a low threat condition increases, the increase in probability of returning to a high threat condition has a dominant effect on the percentage of supplies delivered. However, α also has a quadratic effect that has a 4.5% increase in the percent of supplies delivered by CUAS over the range given in Table 6. This accounts for the compounding effect of having continually low threat conditions. These inter-relations of α and β demonstrate the importance of correctly identifying the threat condition and how likely it is to change. When transitioning between low and high threat conditions, as long as the probability of staying in the say threat condition is about equal, then the percentage of supplies deliveredy by CUAS will be above 20% as evident during the fall season in Table 4.

The threat level of the low threat map ϕ_l has a 10% positive effect on the percentage of supplies delivered by CUAS over the range of the factor. This means that, as the probability of successfully traversing a route from the BSB to COP increases, so does the percentage of supplies delivered by CUAS. The factor ϕ_l also interacts with both α and β. The $\alpha * \phi_l$ two-factor interaction has an effect of 6% over the range of the interaction. Therefore, for a fixed ϕ_l, increasing the probability of staying in a low threat environment increases the effect of successfully traversing a route. Similarly, the $\beta * \phi_l$ two-factor interaction has a -6% effect. When ϕ_l is fixed, as the probability of staying in a high threat environment increases the percentage of supplies delivered by CUAS decreases. Therefore, the net impact of the interactions for a fixed ϕ_l, when considering all other factors in the model, is zero. This explains the results for all seasons from Section 4.3. While the ϕ_{im} varied, the values of α and β really determined the amount of supplies delivered. This further explains why deliveries were attempted in the summer even when conditions were bad. When in a high threat map, the likelihood of conditions improving is so low and, even when a low threat condition is reached, it will not remain. Therefore, the decision to accept the risk of sending on a high threat map is taken, resulting in low percent of supplies delivered.

The CUAS fleet size V affects the percent of supplies delivered by 8%. This represents the effect that the Army has on the percentage of supplies delivered by CUAS. All other factors are exogenous and are elements of the operating environment. The ability to deliver supplies does not depend on *crew* but on the total number of available CUAS. If spare CUAS exist then no matter the value of *crew*, delivery is possible. If V is less than *crew*, then regardless of *crew* only the number of V CUAS are available to send.

The final factor in the model is δ_ϕ. The increasing of δ_ϕ decreases the percentage of supplies delivered by CUAS by 3%. This means, as the difference in probability of successfully traversing a route between low and high threat conditions increases, the percent of delivered supplies decreases. The factor δ_ϕ also has a positive quadratic effect of 6%. This occurs because, as δ_ϕ increases the probability of sending a CUAS on the higher threat day decreases and sending a CUAS on a low threat day becomes more of a certainty. Because of the higher probability of an SS occurance on a low threat day, the likelihood of success increases and a maximum number of CUAS are sent when favorable conditions exist. Furthermore, the quadratic effect of δ_{phi} represents sequential days where the high threat condition is much lower than the low threat condition. When this occurs, the likelihood of a low threat environment decreases and the risk of losing a CUAS in a high threat conditions is accepted in order to maintain inventory at the COP and prevent stock-out.

V. Conclusions and Recommendations

5.1 Conclusion

The deterministic demand MILIRP problem formulation we present provides Army decision-makers a model for analyzing the requirements of a CUAS fleet dedicated to sustainment operations. We demonstrate that a BSB that has two CUAS companies having a combined fleet of 24 CUAS and 12 crew has the ability to resupply an infantry battalion. This means that an entire IBCT can be supplied by six CUAS companies. Even if complete resupply by CUAS is not possible due to extreme threat conditions, as in the summer and fall seasons, the amount of supplies delivered by ground convoy or manned aviation assets can be reduced by 20%. This significantly reduces soldier exposure to enemy attack, increases reliability in supply delivery, and allows more manned aviation assets to fill other operational mission requirements. Our formulation of the MILIRP provides a framework for modeling the capability of resupply by CUAS. In particular, properly assessing the probability of completing a route in varying threat conditions and the probability of staying in and transitioning to different threat conditions is key to modeling the use of CUAS in conducting resupply. This model can be used to evaluate the proposed results of other studies (i.e., General Dynamics Information Technology [12]) and the present results with transparency. The dynamic programming formulation provides clear parameterization and allows for easy assessment of the results.

5.2 Limitations

The current formulation and application only models a single battalion-sized COP. The requirements to supply an entire BCT can be modeled as a multiple of the results for a battalion. However, this requires extrapolation of the results and an increase in the variance regarding any estimates in CUAS fleet size requirements. Furthermore, the design of experiment provides insight into the significant factors and their effects; however, selecting a specific range for the values to use in parameterizing the different factors results in only a portion of the entire design space being investigated by our analysis. Therefore, the percent of supplies delivered by CUAS and insights on the significant factors only apply under the range of conditions modeled.

5.3 Potential Future Research

We consider the case of deterministic demand in our formulation of the MILIRP. This accounts for the normal daily consumption of supplies that does not change. However, in conditions of increased attacks by hostile forces, an increase of supply consumption occurs. Further analysis of the MILIRP should consider a stochastic demand in addition to the deterministic demand we model. We believe a stochastic demand that is conditioned on the threat condition of the AO surrounding the COP is an appropriate approach to modeling this variant of the MILIRP.

Another area of further consideration is the treatment of supplies. In our formulation we consider the supplies as a whole and analyze the total weight that must be delivered to the COP. However, the military considers different categories, classes of supplies. Each class of supply has a different utilization rate, especially when considering stochastic demand. For example, the ammunition, water, and medical supply consumptions change depending on the threat condition and combat operations the unit conducts. Therefore, modeling the demand of each class of supply is another area that requires attention and furhter resolution in modeling.

We apply the dynamic programming model to a single battalion within the BCT. An extension of the model needs to consider multiple battalions. We believe the dynamic programming model we formulate can solve the multiple battalion model exactly. Kleywegt *et al.* [14] state that a dynamic programming model can solve exactly instances that have three to four customers, the number of battalions of interest in the MILIRP. After extending the model to include multiple battalion-sized COPs, separating the battalions into the subordinate elements should be done until the model cannot solve the problem. At this point, an approximate dynamic programming formulation should be implemented to solve an instance of the MILIPR with up to 25 COPs.

Finally, separate analysis is required to determine the actual parameterization of the significant factors in the design of experiment. This will develop the actual response region of interest and enable accurate analysis of the requirements to completely conduct resupply for an BCT by CUAS only. Parameters that require special attention and analysis are α, β, ω_l, ω_m, and ω_h. These parameters address the probability of staying in a low and high threat environment and the probability of successfully traversing a portion of the route. These parameters are exogenous to what the Army can control and have the greatest effect on the ability to deliver supplies successfully.

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Vita

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14. ABSTRACT

The inventory routing problem coordinates inventory management and transportation policies when implementing vendor managed inventory replenishment, the business practice were a vendor monitors the inventory of its customers and determines a strategy to replenish each customer. The United States Army uses vendor managed inventory replenishment during combat situations to manage resupply. The military variant of the stochastic inventory routing problem considers delivery failure due to hostile actions. We formulate a Markov decision process model for the military inventory routing problem, with the objective to determine an optimal unmanned tactical airlift policy for resupplying brigade combat team elements in a combat situation using cargo unmanned aerial systems for delivery. Computational results are presented for the military inventory routing problem with direct deliveries. Results indicate that unmanned aerial systems are capable of performing brigade combat team resupply, given the dynamics of the threat situation. An experimental design is employed to determine the set of factors important in a more general context.

15. SUBJECT TERMS

logistics, dynamic programming, Markov decision process, deterministic inventory routing, unmanned aerial vehicles

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